

Hard SAT Math Worksheet

Name: _____ Date: _____

Circle one answer for each question. Use the blank space for calculations. A calculator is allowed.

1. The equation $x^2 - kx + 48 = 0$ has two positive solutions. One solution is three times the other. What is k ?

A. 8
B. 16
C. 12
D. 48

2. $\sqrt{2x + 9}$ is equal to $x - 3$. What value of x satisfies the equation?

A. 8
B. 3
C. 6
D. 0

3. The graph $y = ax^2 + bx + c$ has vertex $(3, -4)$ and passes through $(0, 14)$. What is $a + b + c$?
- A. 2
 - B. 12
 - C. 4
 - D. 14
4. The line $y = mx + 7$ intersects the parabola $y = x^2 + 3x + 11$ at exactly one point. If m is positive, what is m ?
- A. 4
 - B. 1
 - C. 3
 - D. 7

5. A positive quantity is decreased by $p\%$, then the result is increased by $p\%$, where $0 < p < 100$. The final quantity is 84% of the original. What is p ?
- A. 16
B. 40
C. 32
D. 84
6. For the quadratic function $f(x) = ax^2 + bx + c$, $f(1) = 8$, $f(3) = 26$, and $f(5) = 60$. What is $f(8)$?
- A. 120
B. 133
C. 141
D. 136

7. The equation $x^2 - 10x + k = 0$ has two positive roots whose difference is 4. What is k ?

- A. 9
- B. 16
- C. 21
- D. 24

8. For positive constants a and b , $f(x) = a(b^x)$. If $f(2) = 12$ and $f(5) = 96$, what is $f(0)$?

- A. 2
- B. 3
- C. 4
- D. 6

9. A quadratic f has zeros -2 and 6 . Its minimum value is -32 . What is $f(0)$?

- A. -32
- B. -24
- C. -16
- D. 24

10. For which positive value of k does $x^2 - 6x + k = 2x - k$ have exactly one real solution?

- A. 4
- B. 8
- C. 12
- D. 16

11. A positive quantity is increased by $p\%$ and then increased by $2p\%$. The result is 168% of the original. What is p ?

- A. 10
- B. 20
- C. 30
- D. 40

12. If $x + \frac{1}{x} = 5$ for a nonzero real x , what is $x^2 + \frac{1}{x^2}$?

- A. 15
- B. 21
- C. 23
- D. 25

Answer key

1 B · 2 A · 3 C · 4 D · 5 B · 6 C · 7 C · 8 B · 9 B · 10 B · 11 B · 12 C

1. B: 16

Write the roots as r and $3r$. Their product is 48, so $3r^2 = 48$ and $r = 4$ because both roots are positive. The roots are 4 and 12. Their sum equals k , so $k = 16$.

2. A: 8

Because a square root is nonnegative, x must be at least 3. Squaring gives $2x + 9 = x^2 - 6x + 9$, so $x(x - 8) = 0$. The candidate 0 violates $x \geq 3$. The valid solution is 8: $\sqrt{25}$ equals $8 - 3$.

3. C: 4

Vertex form is $y = a(x - 3)^2 - 4$. Using $(0, 14)$ gives $14 = 9a - 4$, so $a = 2$. Expanding gives $y = 2x^2 - 12x + 14$. Thus $a + b + c = 2 - 12 + 14 = 4$. Equivalently, evaluate y at $x = 1$.

4. D: 7

Equating the expressions for y gives $x^2 + (3 - m)x + 4 = 0$. A single intersection requires $(3 - m)^2 - 16 = 0$. Thus $3 - m$ is 4 or -4 , giving $m = -1$ or 7. The positive value is 7.

5. B: 40

The combined multiplier is $(1 - \frac{p}{100})(1 + \frac{p}{100}) = 1 - \frac{p^2}{10000}$. Set this equal to 0.84: $\frac{p^2}{10000} = 0.16$, so $p^2 = 1600$. Since p is positive, $p = 40$. Checking: 0.60 times 1.40 equals 0.84.

6. C: 141

The x -values are spaced by 2. Subtracting the first two equations gives $8a + 2b = 18$; subtracting the next two gives $16a + 2b = 34$. Hence $8a = 16$, so $a = 2$ and $b = 1$. From $f(1) = 8$, $c = 5$. Therefore $f(8) = 2(64) + 8 + 5 = 141$.

7. C: 21

The roots sum to 10 and differ by 4, so they are 3 and 7. Their product equals the constant k , giving $k = 21$.

8. B: 3

Dividing gives $b^3 = \frac{96}{12} = 8$, so $b = 2$. Then $4a = 12$, hence $a = 3$ and $f(0) = 3$.

9. B: -24

Write $f(x) = a(x + 2)(x - 6)$. The vertex lies midway between the zeros at $x = 2$, where $-16a = -32$, so $a = 2$. Then $f(0) = 2(2)(-6) = -24$.

10. B: 8

Rearrange to $x^2 - 8x + 2k = 0$. A repeated root requires discriminant $64 - 8k = 0$, so $k = 8$.

11. B: 20

Set $(1 + \frac{p}{100})(1 + \frac{2p}{100}) = 1.68$. With $p = 20$ the factors are 1.2 and 1.4, whose product is 1.68. The other algebraic root is negative and is excluded.

12. C: 23

Squaring gives $x^2 + 2 + \frac{1}{x^2} = 25$. Subtract 2 to obtain the requested expression, 23.